Learning Rules for Collective Entity Resolution

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Entity Resolution (ER)

Entity resolution (*ironically goes with different names: deduplication, record linkage, entity matching…*)

- to identify **different constants** representing **the same entity**
- usually in structured/semi-structured data sources, e.g. **database**, knowledge graph

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pid	name	age	phone	allergy
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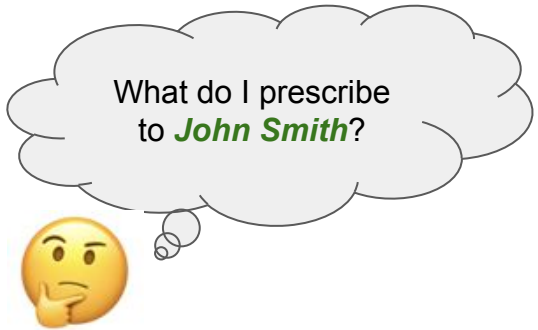
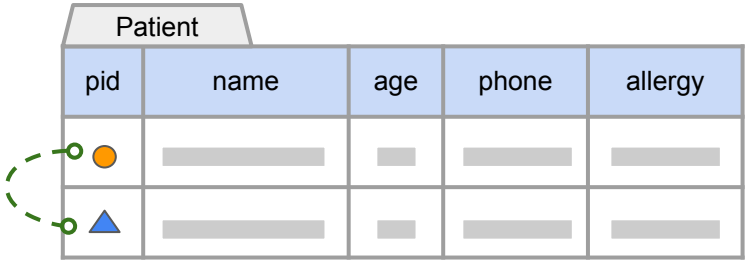


Critical to **data quality** and **decision making**!

Entity Resolution (ER)

Traditional: within **single-table** or **table pair** (same entity type)



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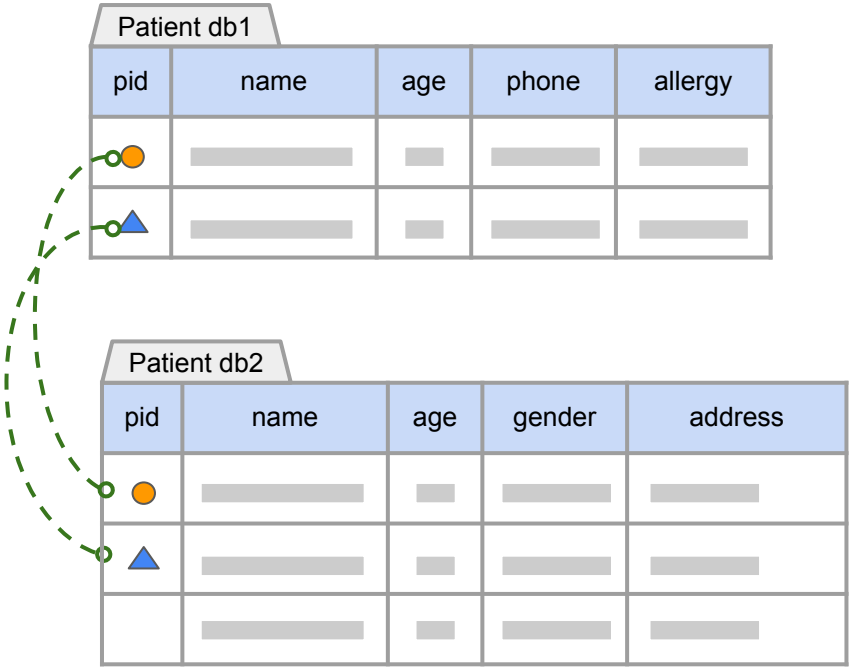

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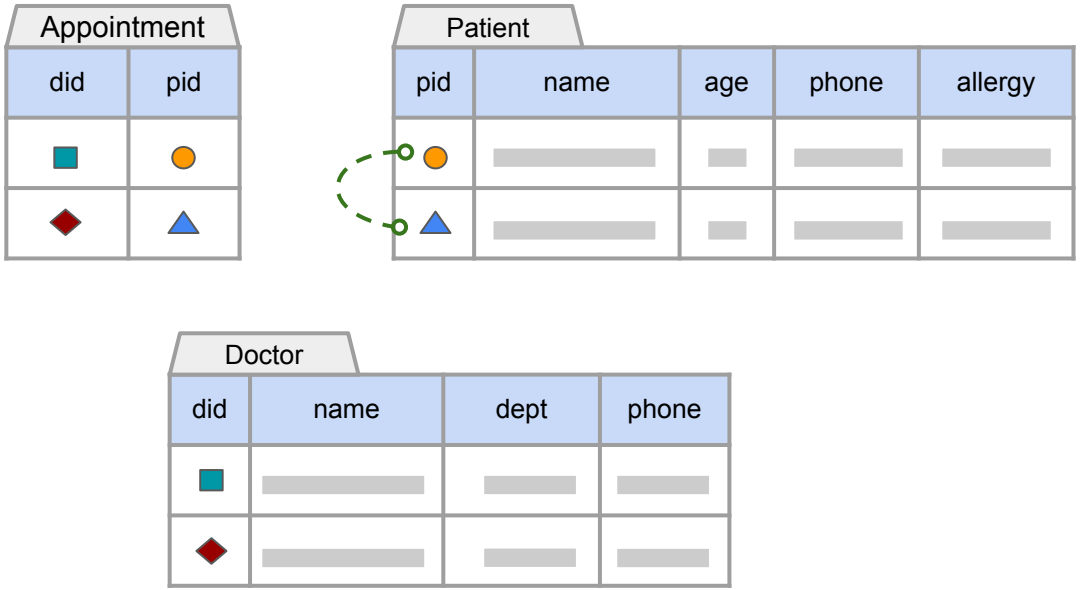

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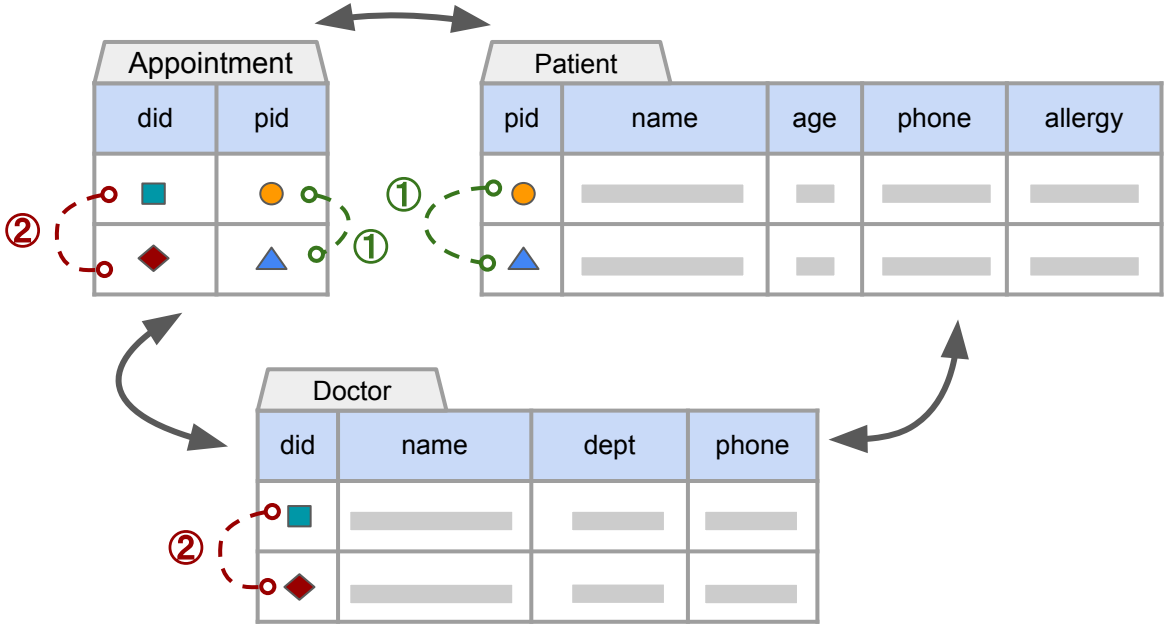
Collective: across **multiple tables**, exploit **inter-dependencies** between tables



Previous work

Neural-based methods excel in pairwise setting, but …

- Failed to capture **transitivity**, predictions may be **inconsistent**
- Not clear how to support **collective ER**
- **Explanation** matters

Previous work

Novel rule-based framework:

- ***LACE (A Logical Approach to Collective Entity Resolution)*** *[Bienvenu et al. 2022]*

ASP-based Implementation:

- ***ASPEn system*** *[Xiang et al. 2024]*

Featuring

- **declarative:** logical **rules** and **constraints**
- **collective:** natural to capture **interdependencies** between tables
- **explainable**: "**why** are *J. Smith* and *John Smith* deemed to be the **same** patient?"

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Expert knowledge is hard to obtain

Rule-based approach

Database D

Appointment

did	pid
d1	p1
d2	p2
d3	p3
d3	p4

Patient

pid	name	age	phone	allergy
p1	J.Smith	32	12345	brupen
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Doctor

did	name	dept	phone
d1	ブラックジャック	surgeon	66677
d2	Black jack	surgeon	66677
d3	Tezuka	skin	77333

ER rules L $x[pid]=y[pid] <= Patient(x), Patient(y), x[name]\approx y[name], x[age,phone]=y[age,phone]$

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If … then … are the same

pid of two patients are the same if they have **similar names**, and **identical age and phone number**

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⇕ If … then … are the same

pid of two patients are the same if they have **similar names**, and **identical age and phone number**

⇕ database query

```
select x.pid, y.pid from patient as x, patient as y
        where x.name ≈ y.name, x.age = y.age, x.phone = y.phone
```

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Query answers

result

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Learning ER rule

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ER rules L

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Learning ER rule $\sqsubseteq$ Query reverse engineering

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ER rules L

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Searching for the **rules (queries)** give rise to the **ER answers** on a **database instance**

Research aim and questions

Learning ER rules $\sqsubseteq$ Query reverse engineering (QRE) $\sqsubseteq$ Inductive logic programming (ILP)

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General goal:
taking inspirations from **ILP** and **QRE** to develop a **theoretically sound** method on **learning collective ER rules**

Research aim and questions

Learning ER rules ⊑ Query reverse engineering (QRE) ⊑ Inductive logic programming (ILP)

General goal:
taking inspirations from **ILP** and **QRE** to develop a **theoretically sound** method on **learning collective ER rules**

Searching for the **rules (queries)** give rise to the **ER answers** on a **database instance**

1. What **language**?
2. What is an **ER answer** under such language?
3. What are the **structure** of interest?
4. How can we **characterise** the structure to **facilitate searching**?

The language

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Rule language: tuple relational calculus + LACE framework

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Rule language: **tuple relational calculus** + LACE framework

Schema $\mathcal{S} = \{Doctor[did, name, dept, phone], \ldots, Appointment[did, pid]\}$

order invariant

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Tuple E.g. *{pid: J.Smith, age: 32, phone:12345, allergy: brupen}*

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$$x[pid]=y[pid] <= Patient(x), Patient(y), x[name]\approx y[name], x[age,phone]=y[age,phone]$$

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$x[pid]=y[pid] <= Patient(x), Patient(y), x[name] \approx y[name], x[age,phone]=y[age,phone]$

Soft rule: **less certain** conditions for **likely** merges

$x[name]=y[name] <\sim Doctor(x), Doctor(y), App(w), App(z), x[did]=w[did],$
$y[did]=z[did], w[pid]=z[pid], x[dept,phone]=y[dept,phone]$

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Previously $EqV(<i,2>,<i',2>)$

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Constraint: enforce consistency

$\perp <= Patient(x), Patient(y), x[pid]=y[pid], x[name] \neq y[name]$

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EqRel E $E = \{\}$

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Constraint: $\bot \leftarrow Patient(x), Patient(y), x[pid]=y[pid], x[name] \neq y[name]$

EqRel E $\quad E = \{(p1,p2), (p3,p4), (d1,d2)\} \longrightarrow$ A *solution* is an E s.t. D_E satisfies **H** and **C**

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Soft rule: $x[did]=y[did] <= \ldots y[did,pid], y[did,pid]=z[did,\ldots$

Observation: ER rule only talks about **conditions** are **relevant to the head**

Connectivity

Definition 1: *Connected rule*

An ER rule is **connected** iff there does not exist disjoint variables in its rule body.

- x and y are the same tuple variable
- if $x \neq y$ and there exists an atom(s) $x[A] \otimes y[B]$
- there exists atoms $x[A] = c$ and $y[B] = c$
- there exists a tuple variable z, s.t. x is connected to z and z is connected to y

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Ensure all body conditions **are relevant** and **at least one comparison**

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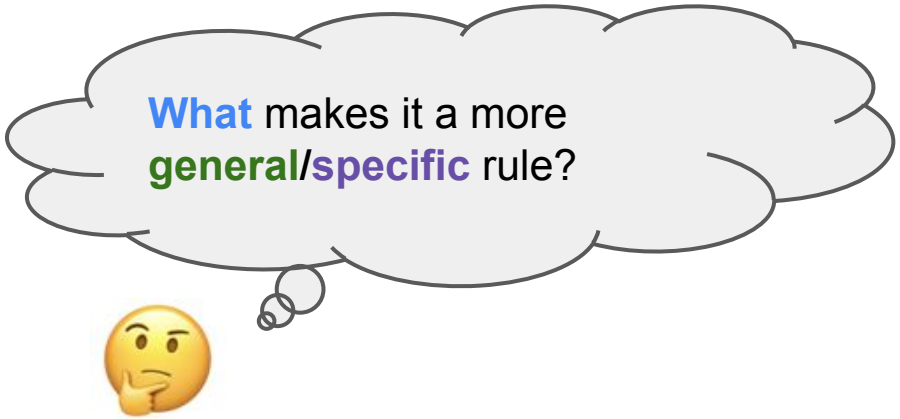
Connected: *x[did]=y[did] <~Doctor(x), Doctor(y), App(w), App(z), x[did]=w[did], y[did]=z[did], w[pid]=z[pid], x[dept,phone]=y[dept,phone]*

Not Connected:

x[did]=y[did] <~Doctor(x), Doctor(y), Patient(w), Patient(z), w[name]≈z[name].

x[did]=y[did] <~Doctor(x), Doctor(y), App(w), App(z), x[did]=w[did], y[did]=z[did] .

Subsumption order



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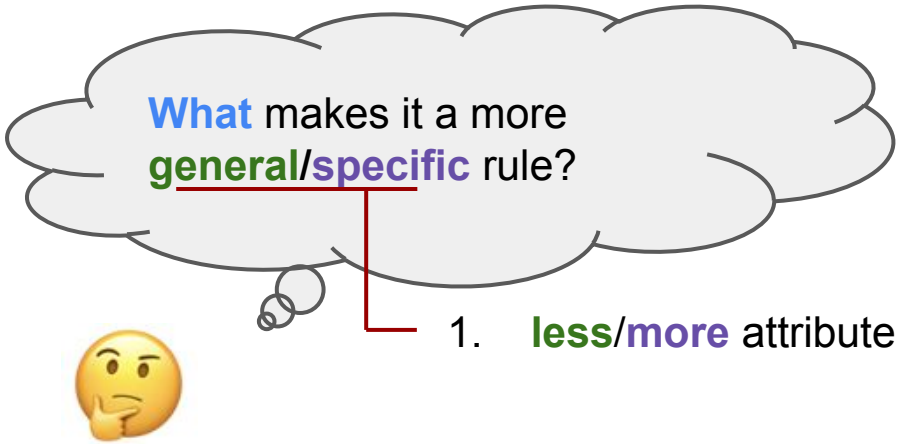

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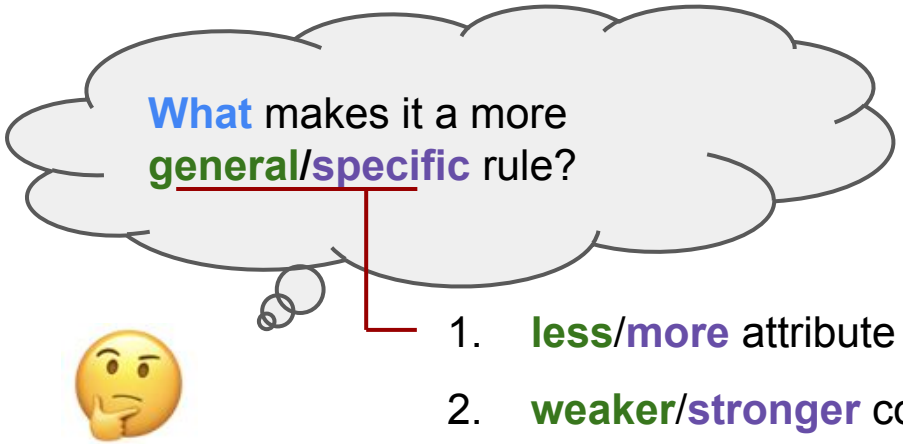

1. **less**/**more** attribute comparisons
2. **weaker**/**stronger** comparisons applied

Subsumption order

Example: Consider following ER rules over schema $\mathcal{S}=\{R[A,B]\}$

$r_1: \; x[A] = y[A] \leftarrow R(x), R(y), x[B] = y[B].$

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From previous intuition, it should be the case that $r_3 \succ r_2 \succ r_1$

But r_3 is actually **the same** rule as r_1 ?

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But r_3 is actually **the same** rule as r_1 ?

In **ILP**, subsumption can be defined as **substitution**

Subsumption order

Definition 2: *subsumption order* $\geq$

$r' \geq r$ (more specific) if there exist a mapping $g : var(r) \rightarrow var(r')$, s.t. the rule r_g satisfies:

- *head*(r_g) = *head*(r')
- Every relational atom $a \in$ *body*(r_g) is in *body*(r')
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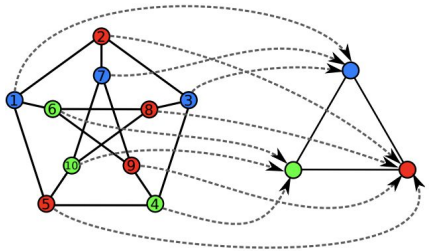
In fact, this also aligns with **homomorphism** theorem in **database theory.**

r' is subsumed by r ⇔

existences of mapping g ⇔

answers of (body query of) r' is **contained by** answers of r



Subsumption order

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Fixing a rule r_h. Let Γ be the set of all possible **connected** rules built from r_h. Then the subsumption ordered set **$<\Gamma, \geq>$ is a lattice**.

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glb (least generalisation) and **lub (greatest specialisation)**

exist for any **finite set** of rules built from r_h

Refinement operator

An ***(upward) refinement operator***
over the quasi-ordered space $\langle \Gamma, \succeq \rangle$ is a function ρ, s.t. $\rho(r) \subseteq \{r' | r' \succeq r\}$, for every $r \in \Gamma$.

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Refinements $\rho^*(r) = \rho_1(r) \cup \ldots \cup \rho_n(r)$.

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Desired properties

- **locally finite**: each application results to a finite set.
- **complete**: for every r' s.t. $r' \succ r$, there exists $r^* \in \rho^*(r)$, s.t. $r^* \sim r'$.
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Given r, we consider the 1-step refinements $\rho(r)$ as follows:

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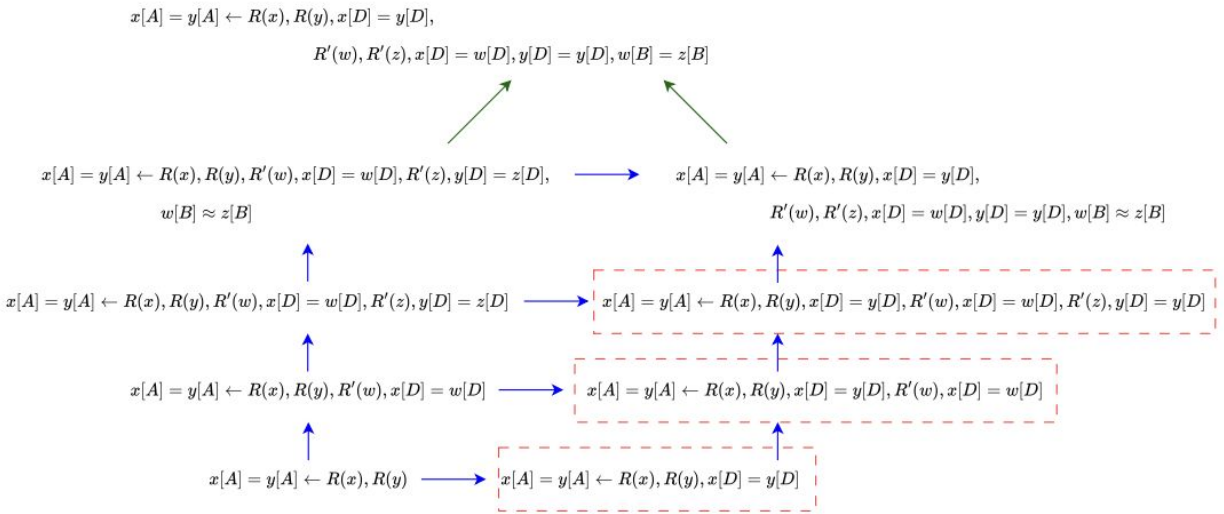
Refinement operator for ER rules



Refinements example to merge $R[A]$ on schema $\mathcal{S} = \{R[A, D], R'[D, B]\}$

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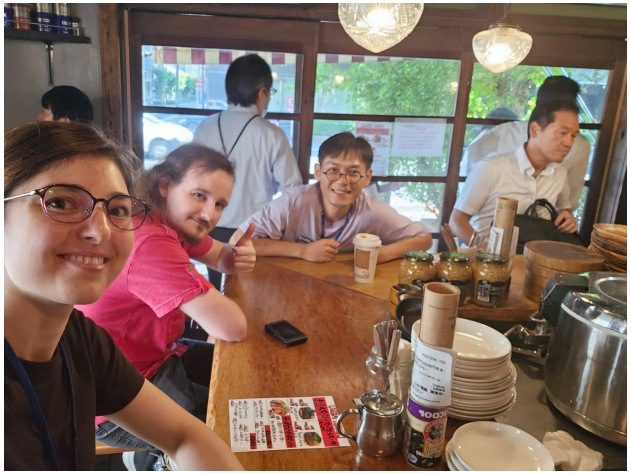
(at a glance) if consider only **finite set of constants**, our ρ is **locally finite**, and **complete**, though **not proper**.

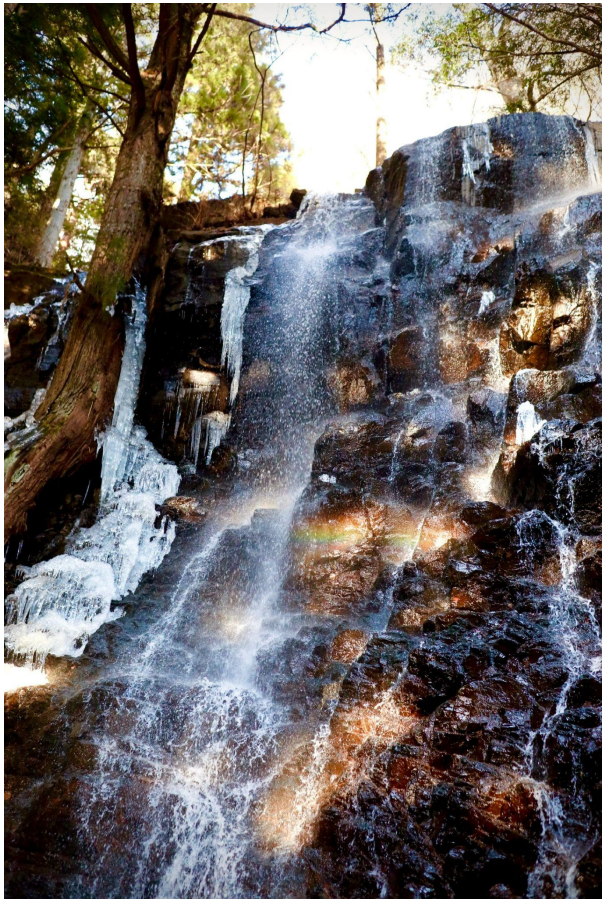
Conclusion

- We study **learning ER rules** under **LACE** framework using **tuple relational calculus**.
- We defined **connectivity** of ER rules restricting conditions to be **relevant**.
- We defined **subsumption ordering search space** of all possible rules, which induces **a lattice structure** and **guarantee** that if one rule is **more general** than the other, then **set containment holds** for **answer sets** of the rules.
- We defined an **upward refinement operator** for navigating the subsumption ordered rule space, and showed that such operator is **locally finite** and **complete**, though **not proper**.

Ongoing & future work

- Characterise **hard** and **soft** rules under a subsumption ordered rule set
- **Learning algorithm** taking into account **LACE semantics**:
  - Meta program *[Inoue et al. 2019 IJCAI]*
  - Fitting Algorithms for Conjunctive Queries *[ten Cate et al. 2024 SIGMOD]*
- Experiments





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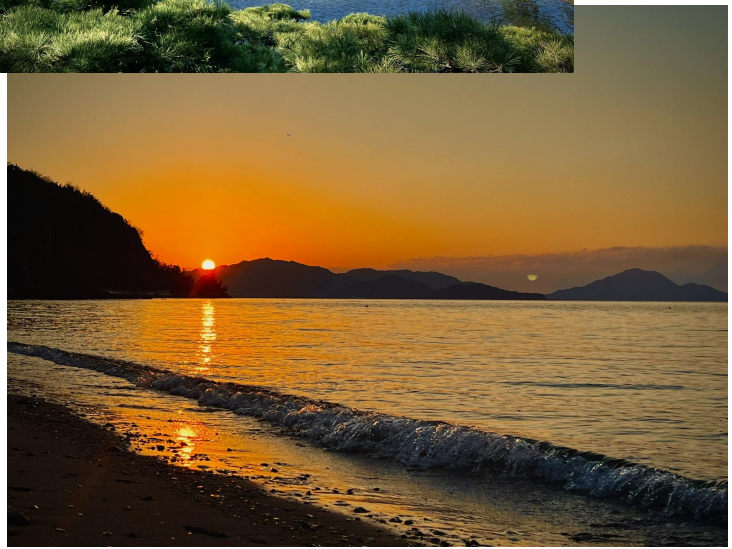

高松&男木島

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